Foundations of Query Languages

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Measurement of Query Processing

- Complexity classes are usually defined for Decision (yes/no) problems.
- Queries may have a large output.
- It would be unfair to count the size of the output as *complexity*.
- We therefore consider the following decision problems, which areall computationally equivalent purposes (logspace equivalent).
  - Boolean Queries $D \models Q$?
  - The Query of Tuple problem (QOT)$t \in Q(D)$?
  - The Query-Emptiness problem $Q(D) \neq \emptyset$

Measurement of Query Processing

- Data Complexity $Q(D)$: where D is considered as input, while Q as fixed.
- Query Complexity $Q(D)$: where Q is considered as input, while D as fixed.
- Combined Complexity $Q(D)$: both D and Q are considered as input.

QBF Problem

QBF (Quantified Boolean Formulas) (a.k.a. QSAT)

$$Q_1 x_1 Q_2 x_2 \dots Q_n x_n \phi(x_1, x_2, \dots, x_n)$$

$Q_i \in \{\exists, \forall\}$ is a quantifier, ϕ is a Boolean formula in CNF; $Q_1 \dots Q_n$ alternates between $\exists$ and $\forall$.
For instance, $\exists x_1 \forall x_2 \dots \exists x_n$

QBF Example

$$\exists x_1 \forall x_2 \exists x_3 (x_1 \vee x_2 \vee x_3) \wedge (x_2 \vee \neg x3) \wedge (x_1 \vee \neg x_2 \vee \neg x_3) \wedge (\neg x_1 \vee x_2)$$

$$\exists x_1 \forall x_2 \exists x_3 (x_1 \vee x_2 \vee \neg x_3) \wedge (x_2 \vee x3) \wedge (x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_1 \vee \neg x_2 \vee \neg x_3)$$

QBF Problem - Algorithm

Algo *Truth* (Φ)

if Φ is quantifier-free **then** return its truth value

end if

Let $\Phi = Q_1 x_1 \ldots Q_n x_n \, \phi(x_1, \ldots, x_n)$;

$b_0 = \mathit{Truth}(Q_2 x_2 \ldots Q_n x_n \, \phi(0, x_2, \ldots, x_n))$;

$b_1 = \mathit{Truth}(Q_2 x_2 \ldots Q_n x_n \, \phi(1, x_2, \ldots, x_n))$; % re-using space

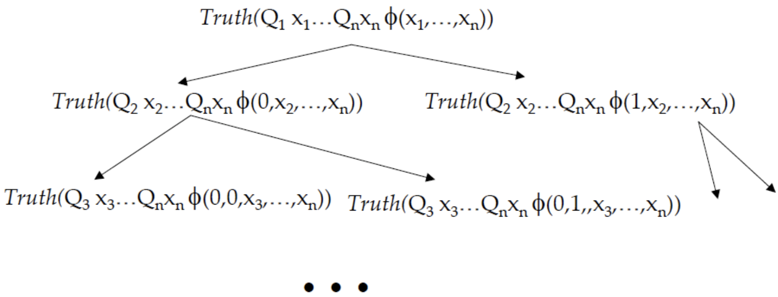
if $Q_1 = \exists$ **then** return $b_0 \vee b_1$

else return $b_0 \wedge b_1$

end if

end Algo

QBF Problem - Algorithm



Depth of recursion: n, at each step the stack size is $poly(n)$
$\rightarrow$ QBF in PSPACE

Logspace Transducer

A logspace transducer is a Turing machine with a read-only input tape, a write-only output tape, and a read/write work tape.

- The work tape may contain $O(\log n)$ symbols.
- A logspace transducer M computes a function $f : \Sigma^* \longrightarrow \Sigma*$, where $f(w)$ is the string remaining on the output tape after M halts when it is started with w on its input tape.
- f is a logspace computable function

Logspace computation

Some capabilities of logspace machines

- Maintain (a constant number) of counters on the worktape, and increment or decrement such counter
- Locate particular items of a well-structured input (integers, list item) via pointers consisting of input-tape address
- Access, process and compare input items in a bit-by-bit fashion

Binary integer addition, subtraction, comparing data items or lists, copying data items, searching, sorting, etc.

Language in Logspace

$$A = \{0^k 1^k | k \geq 0\}$$

is a member of Logspace.

- on the work tape, maintain a counter C, and one pointer P to the input tape
- Read the content of what P is pointing to; increase the counter as long as the content is 0 (move P to right)
- As soon as the first 1 is met, start decreasing the counter
- Accept if read the end and the counter is set to 0, reject otherwise

Join Operation in Logspace

Join of two relations r and s on attribute A

- Maintain two tuple pointers τ_r and τ_s
- Outer loop: move τ_r to point successively to each tuple of r
- For each such tuple t_r, an inner loop makes τ_s successively point to each tuple t_s
- For each combination of tuples t_r and t_s, identify the fields corresponding to attribute A and check, bit by bit, where the A-value of t_r and t_s are equal
- If yes, the relevant part of t_r and t_s are (bitwise) copied to the output tape
- Space used on work tape: 4 pointers + 2 + a constant number of counters

Complexity of FO Queries

Theorem

Evaluating Boolean FO (or RA) queries is as follows:

- *PSPACE complete (combined complexity)*
- *PSPACE complete (query complexity)*
- *in LOGSPACE (data complexity)*

The same complexity results apply to the Query-emptiness problem and to the QOT-Problem

FO Evaluation

Eval(I, φ)
Case
  φ is $p(t_1,\ldots,t_k)$: return $p^I(t_1,\ldots,t_k)$
  φ is $\neg\,\psi$: return $\neg$ Eval(I,ψ)
  φ is $\theta \wedge \psi$: return Eval(I,θ) $\wedge$ Eval(I,ψ)
  φ is $(\exists x)\psi$:
    B:= false
    for $a \in$ **dom** do
      B := B $\vee$ Eval(I, $\varphi_{x \to a}$)
    return B
End Case

FO Complexity

$$m(\log m + m \log n)$$

- Combined complexity (both m and n as input):
  $m(\log m + m \log n) \rightarrow$ PSPACE
- Data complexity (n as input, m as constant):
  $\log n \rightarrow$ LOGSPACE
- Query complexity: (m as input, n as constant) :
  $m^2 \rightarrow$ PSPACE

FO Complexity: Combined Complexity

PSPACE hardness proof: polynomial reduction from QBF problem (PSPACE complete)

- dom: $\{1, 0\}$
- Database: true(1), false(0)
- $\exists x_1 \forall x_2 \exists x_3 (x_1 \vee \neg x_2 \vee x_3) \wedge \ldots$
  $\exists x_1 \forall x_2 \exists x_3 (\mathit{true}(x_1) \vee \neg \mathit{false}(x_2) \vee \mathit{true}(x_3)) \wedge \ldots$

Data Complexity

Data complexity (n as input, m as constant): $\log n \longrightarrow$ in Logspace
Can we get a better upper bound?

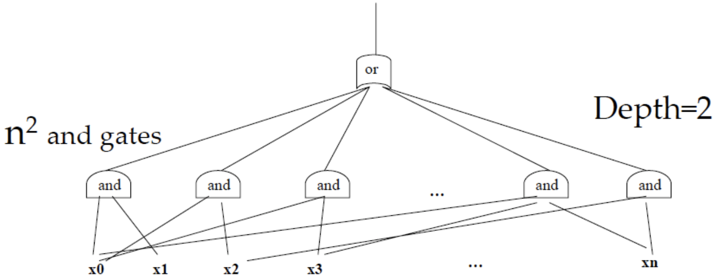
$$AC0 \subset NC1 \subseteq L \subseteq NL \subseteq LOGCFL \subseteq NC2$$

We first need to define circuit complexity classes.

Boolean Circuits

- size= # gates
- depth= longest path from input to output
- formula(or expression): graph is a tree
- can build circuit family that decides L

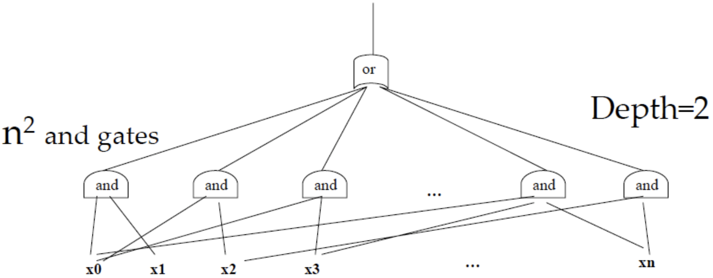
Circuit Family



What is the function of this circuit?

It accepts the language that consists of strings with at least two 1's.

Circuit Family



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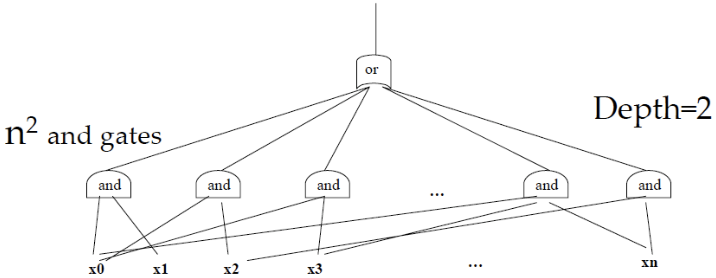

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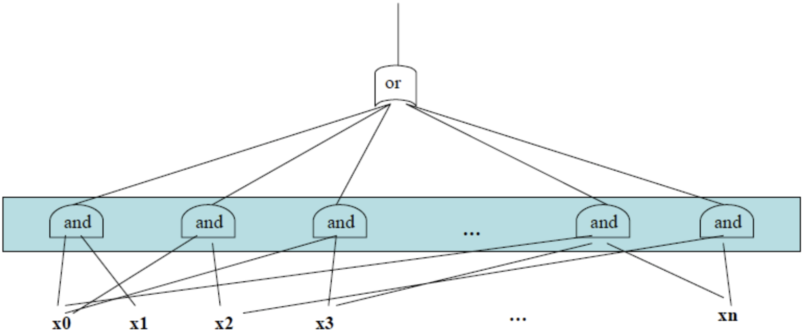

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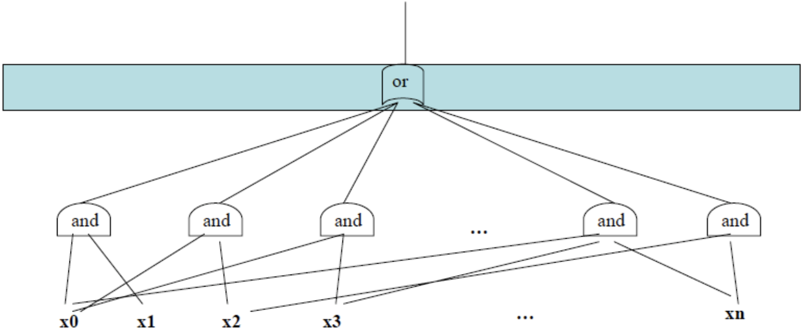
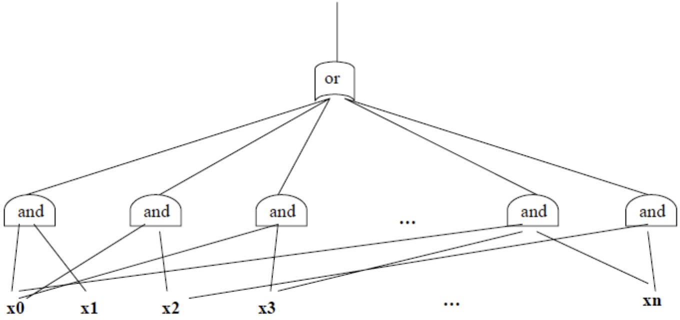


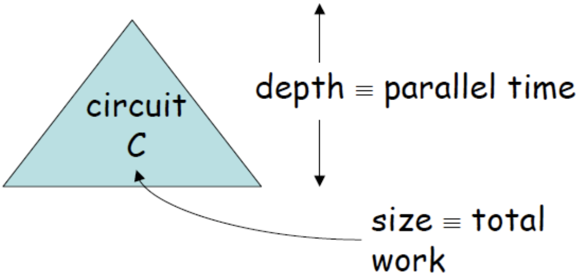
Circuit Family



2 steps, $O(n)$ processors

Parallelism

uniform circuits allow refinement of polynomial time:



Small Depth Circuit

A small depth circuit is a polynomial-size circuit whose depth is poly-logarithmic in its size. That is: a circuit with
size=poly(n) and depth=$O(\log^k n)$
Such circuits capture the notion of efficient parallel computation.

NC and AC

- $NC = \bigcup_k NC^k$ (Nick's Class) class of languages decided by families of fan-in 2 circuits $\{C_n\}$ s.t.
  size(C_n)=poly(n) and depth(C_n)=$O(\log^k n)$.
- $AC = \bigcup_k AC^k$, defined analogously, difference: arbitrary fan-in allowed.

Examples $\Sigma = \{0, 1\}$

- $1 \cdot \Sigma^* \cdot 1 \in NC0$
- $\{0^k 1^k (k \geq 0)\} \in AC0$
- PARITY $:= \{w \in \Sigma^* | \#1(w) odd\} \in NC1$

Data Complexity

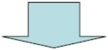
Theorem

Data complexity of FO query: Complete for logtime uniform AC0.

$$\pi_C(\sigma_{A=B}(R_1 \times R_2)) - R_1$$

R_1	A
	a
	b

R_2	B	C
	a	b
	a	c
	b	c



$R_1(a)$	$R_1(b)$	$R_1(c)$	$R_2(a,a)$	$R_2(a,b)$	$R_2(a,c)$	$R_2(b,c)$
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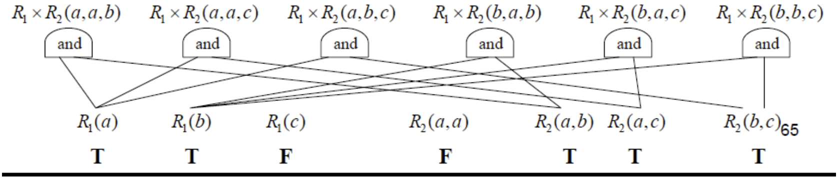
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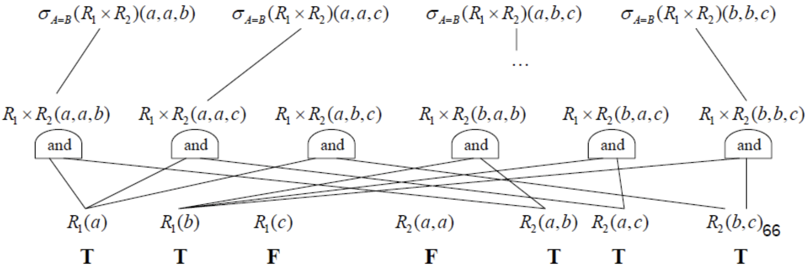
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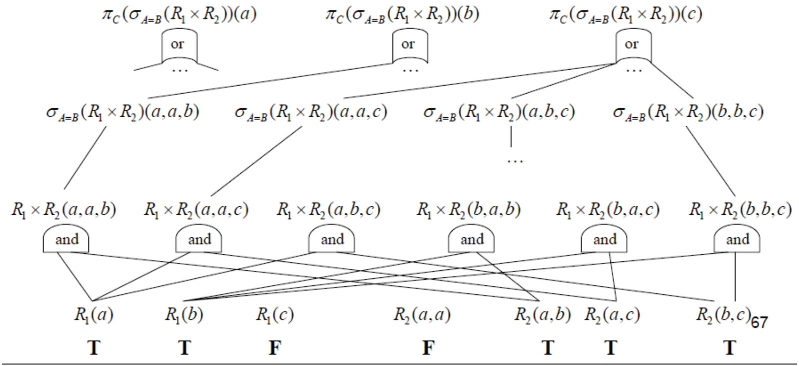
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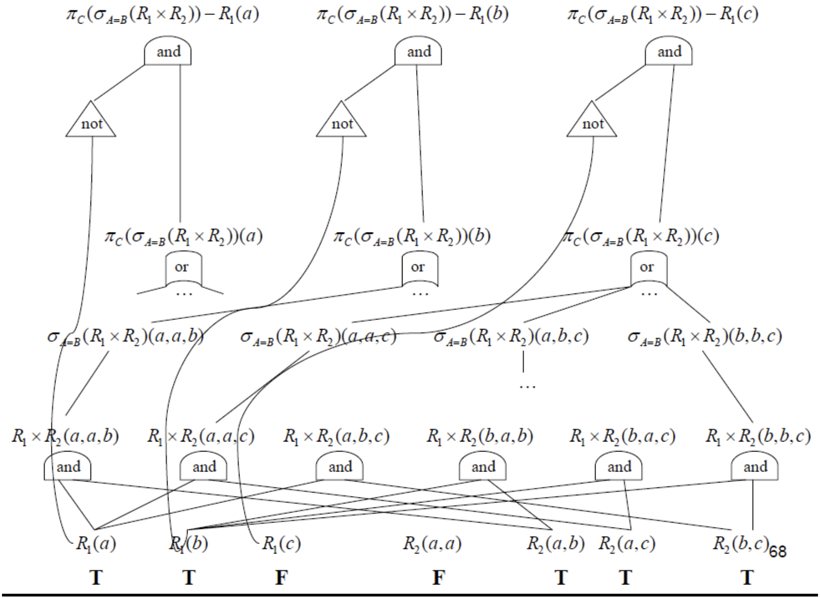
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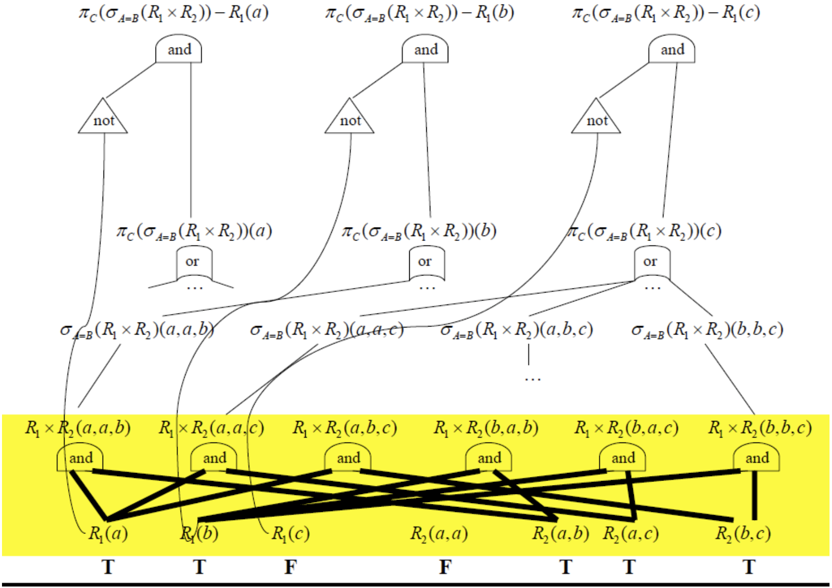
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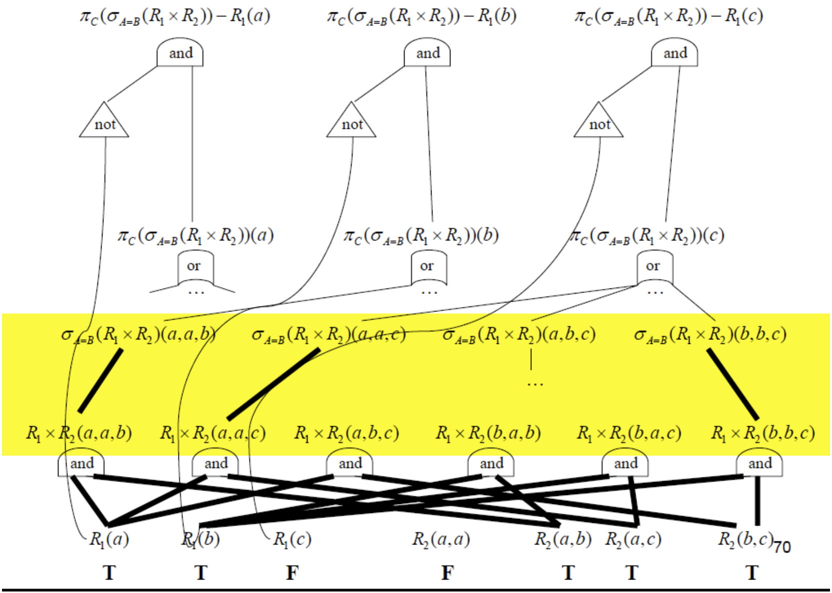
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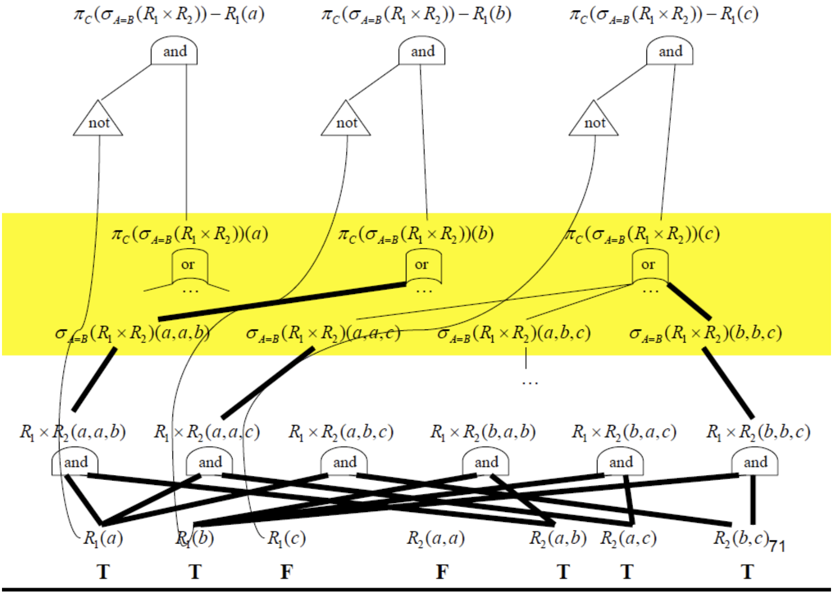
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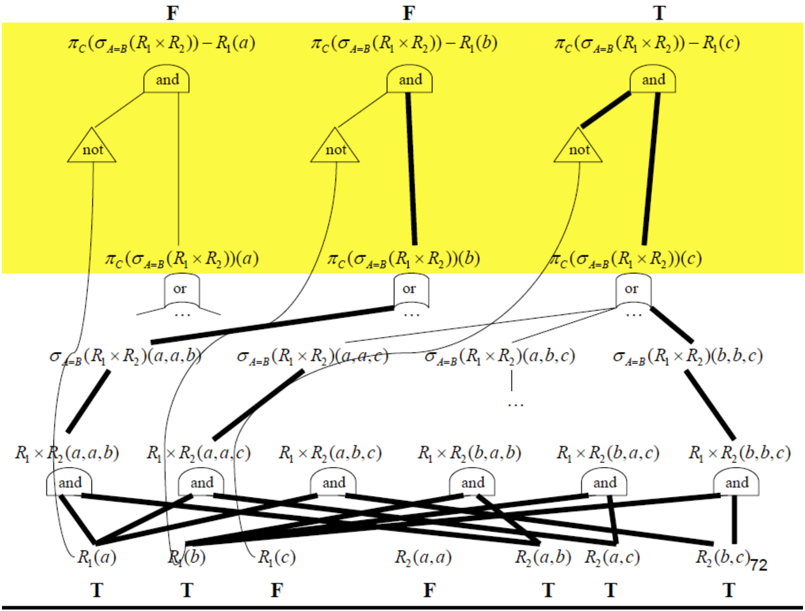
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Remarks

- Schema and query are assumed fixed.
- Database and size of the active domain are variable.
- Uniform families of gates:
  Total number of input gates uniquely determines size of active domain.
- Example:Schema: $R(ABC), S(D), T(EF)$
  Number of input gates: $n^3 + n + n^2$ for some n (which is the size of the active domain).